ON THE INTERRELATION BETWEEN TEMPERATURES FOR NONLINEAR HEAT-TRANSFER PHENOMENA IN HEAT LIBERATING ELEMENTS

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Relationships between temperatures at various points of a heat liberating element are established.

The temperature on the surface of a heat liberating element, i.e., a radio resistor, condensor, coil, etc., is usually determined more simply from experiment or computation. It is interesting to establish a correlation between the temperatures at various points of a heat liberating element, particularly between the temperature at surface points and points within the bulk.

Let us consider a system of nonlinear differential equations describing heat transfer in a body with an energy source (sink) of time-invariant intensity:

$$C(\theta)\frac{\partial\theta}{\partial \mathrm{Fo}} = \operatorname{div}[L(\theta)\operatorname{grad}\theta] + \mathrm{Po}, \tag{1}$$

$$-[L(\theta)\operatorname{grad}\theta]_s = pf(\theta_s), \tag{2}$$

$$\theta_{\mathrm{Fo}=0} = F. \tag{3}$$

Let us construct the solution of this system in the form of a function ϑ which reproduces the law of external heat exchange

$$\vartheta = f(\theta).$$

On the other hand, in a given range of temperature variation, and if this range is relatively large, then the thermal parameters $C(\theta)$ and $L(\theta)$ in its individual sections can be approximated by the equations

$$C(\theta) = a_1\frac{\partial f}{\partial\theta}, \tag{4}$$

$$L(\theta) = a_2\frac{\partial f}{\partial\theta}. \tag{5}$$

Substitution of (4), (5) reduces (1)-(3) to a system of convective heat transfer for the new variable ϑ:

$$\frac{a_1}{a_2}\frac{\partial\vartheta}{\partial \mathrm{Fo}} = \nabla^2\vartheta + \frac{1}{a_2}\mathrm{Po}, \tag{6}$$

$$-(\operatorname{grad}\vartheta)_s = \frac{p}{a_2}\vartheta_s, \tag{7}$$

$$\vartheta_{\mathrm{Fo}=0} = \psi. \tag{8}$$

The solution (6)-(8) can be written as [1]

$$\vartheta(X, Y, Z, \mathrm{Fo}) = \vartheta_{\mathrm{lim}}(X, Y, Z) - W(X, Y, Z, \mathrm{Fo}). \tag{9}$$

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where the $\vartheta_{\lim}$ (X, Y, Z) in (9) is the limit value of the function ϑ which holds for Fo = ∞, and

$$W(X, Y, Z, \mathrm{Fo}) = \sum_{i=0}^{\infty} A_i U_i(X) U_i(Y) U_i(Z) \exp(-m_i \mathrm{Fo}) \tag{10}$$

is a function of the time and the coordinates which tends to zero as Fo → ∞.

Here $m_0 = m$, m_1, m_2, . . . is a number of increasing positive constants

$$0 < m_0 < m_1 < m_2 < \ldots \tag{11}$$

and the functions U_i depend only on the coordinates of points of the body.

Because of the inequality (11) the series in the right side of (10) converges rapidly. Hence, even for times not very close to the beginning, all terms except the first can be neglected therein

$$W(X, Y, Z, \mathrm{Fo}) = AU(X) U(Y) U(Z) \exp(-m \mathrm{Fo}). \tag{12}$$

Having been given some fixed values $Y = Y_\Phi$ and $Z = Z_\Phi$, as is done in [2], we find from (12)

$$U(X) = \frac{W(X, Y_\Phi, Z_\Phi, \mathrm{Fo})}{AU(Y_\Phi) U(Z_\Phi) \exp(-m \mathrm{Fo})}.$$

Correspondingly, for some fixed values X_Φ, Z_Φ and X_Φ, Y_Φ we have

$$U(Y) = \frac{W(X_\Phi, Y, Z_\Phi, \mathrm{Fo})}{AU(X_\Phi) U(Z_\Phi) \exp(-m \mathrm{Fo})} \text{ and } U(Z) = \frac{W(X_\Phi, Y_\Phi, Z, \mathrm{Fo})}{AU(X_\Phi) U(Y_\Phi) \exp(-m \mathrm{Fo})}.$$

Substituting U(X), U(Y), U(Z) into (12) yields the general nature of the correlation between the functions W

$$W(X, Y, Z, \mathrm{Fo}) = \frac{W(X, Y_\Phi, Z_\Phi, \mathrm{Fo}) W(X_\Phi, Y, Z_\Phi, \mathrm{Fo}) W(X_\Phi, Y_\Phi, Z, \mathrm{Fo})}{W^2(X_\Phi, Y_\Phi, Z_\Phi, \mathrm{Fo})}.$$

Taking account of (9) this relationship is written for the function ϑ as

$$\begin{aligned}\vartheta_{\lim}(X, Y, Z) - \vartheta(X, Y, Z, \mathrm{Fo}) &= [\vartheta_{\lim}(X, Y_\Phi, Z_\Phi) - \vartheta(X, Y_\Phi, Z_\Phi, \mathrm{Fo})] \\ &\times [\vartheta_{\lim}(X_\Phi, Y, Z_\Phi) - \vartheta(X_\Phi, Y, Z_\Phi, \mathrm{Fo})] [\vartheta_{\lim}(X_\Phi, Y_\Phi, Z) \\ &- \vartheta(X_\Phi, Y_\Phi, Z, \mathrm{Fo})] [\vartheta_{\lim}(X_\Phi, Y_\Phi, Z_\Phi) - \vartheta(X_\Phi, Y_\Phi, Z_\Phi, \mathrm{Fo})]^{-2}.\end{aligned} \tag{13}$$

In the limit as Fo = ∞, i.e., in the steady state, the relationship (13) simplifies to

$$\vartheta_{\lim}(X, Y, Z) = \frac{\vartheta_{\lim}(X, Y_\Phi, Z_\Phi) \vartheta_{\lim}(X_\Phi, Y, Z_\Phi) \vartheta(X_\Phi, Y_\Phi, Z)}{\vartheta_{\lim}^2(X_\Phi, Y_\Phi, Z_\Phi)}. \tag{14}$$

Particular dependences

$$\begin{aligned}\vartheta_{\lim}(X, Y, Z) - \vartheta(X, Y, Z, \mathrm{Fo}) &= [\vartheta_{\lim}(X, Y_s, Z_s) \\ &- \vartheta(X, Y_s, Z_s, \mathrm{Fo})] [\vartheta_{\lim}(X_s, Y, Z_s) - \vartheta(X_s, Y, Z_s, \mathrm{Fo})] \\ \times [\vartheta_{\lim}(X_s, Y_s, Z) - \vartheta(X_s, Y_s, Z, \mathrm{Fo})] &[\vartheta_{\lim}(X_s, Y_s, Z_s) - \vartheta(X_s, Y_s, Z_s, \mathrm{Fo})]^{-2}\end{aligned} \tag{13'}$$

can be obtained from the general dependences (13), (14), and correspondingly for Fo = ∞

$$\vartheta_{\lim}(X, Y, Z) = \frac{\vartheta_{\lim}(X, Y_s, Z_s) \vartheta_{\lim}(X_s, Y, Z_s) \vartheta(X_{\text{II}}, Y_s, Z)}{\vartheta_{\lim}^2(X_s, Y_s, Z_s)}. \tag{14'}$$

The relationships (13'), (14') permit computation of the functions ϑ for points in the bulk of a heat liberating element, and also the temperatures θ for a given law of external heat exchange $f(\theta)$ if the surface temperatures are known at corresponding points.

Heat liberating elements most often operate in the radiant convective heat-exchange mode

$$-[L(\theta) \operatorname{grad} \theta]_s = (\mathrm{Bi} + \mathrm{G}) \left[\frac{\mathrm{Bi}}{\mathrm{Bi} + \mathrm{G}} (\theta_s - 1) + \frac{\mathrm{G}}{\mathrm{Bi} + \mathrm{G}} (\theta_s^4 - 1) \right]. \tag{15}$$

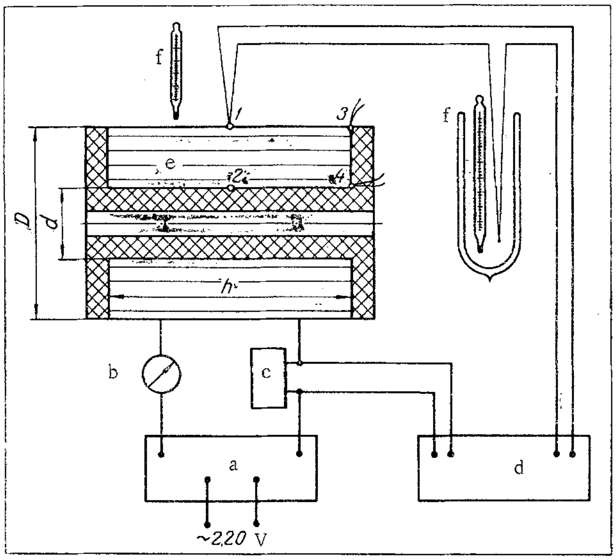

Fig. 1. Test diagram: a) stabilized rectifier VS-12; b) milliammeter; c) resistance box R-314; d) potentiometer R-314; e) range multipliers; f) thermometers; 1, 2, 3, 4) thermocouples.

Following the method elucidated here, let us construct the solution of the system (1), (15), (3) in the form

$$\vartheta = \frac{\mathrm{Bi}}{\mathrm{Bi}+\mathrm{G}}(\theta - 1) + \frac{\mathrm{G}}{\mathrm{Bi}+\mathrm{G}}(\theta^4 - 1). \qquad (16)$$

Substitution of the expression (16) for ϑ into the relationship (13) permits expressing it in terms of the relative temperatures θ and the ratio Bi/G. Further manipulation reduces to the following. The factor with power -2 in the right side is transferred to the left. The left and right sides of the relationship are written as algebraic equations in powers of Bi/G.

Equating the expressions with identical powers of Bi/G, we obtain four equations connecting θ at corresponding points of the heat liberating element which do not contain Bi/G. Of these four equations, the relationship between the excess heats $\Delta T = T - T_c$ of corresponding points of the heat liberating element, which has the form

$$\begin{aligned}\Delta T_{\mathrm{lim}}(X, Y, Z) - \Delta T(X, Y, Z, \mathrm{Fo}) = [\Delta T_{\mathrm{lim}}(X, Y_\Phi, Z_\Phi) \\ - \Delta T(X, Y_\Phi, Z_\Phi, \mathrm{Fo})]\,[\Delta T_{\mathrm{lim}}(X_\Phi, Y, Z_\Phi) - \Delta T(X_\Phi, Y, Z_\Phi, \mathrm{Fo})] \\ \times [\Delta T_{\mathrm{lim}}(X_\Phi, Y_\Phi, Z) - \Delta T(X_\Phi, Y_\Phi, Z, \mathrm{Fo})] \\ \times [\Delta T_{\mathrm{lim}}(X_\Phi, Y_\Phi, Z_\Phi) - \Delta T(X_\Phi, Y_\Phi, Z_\Phi, \mathrm{Fo})]^{-2}. \qquad (17)\end{aligned}$$

is most acceptable for practical utilization. Correspondingly, for Fo = ∞

$$\Delta T_{\mathrm{lim}}(X, Y, Z) = \frac{\Delta T_{\mathrm{lim}}(X, Y_\Phi, Z_\Phi)\,\Delta T_{\mathrm{lim}}(X_\Phi, Y, Z_\Phi)\,\Delta T_{\mathrm{lim}}(X_\Phi, Y_\Phi, Z)}{\Delta T^2_{\mathrm{lim}}(X_\Phi, Y_\Phi, Z_\Phi)}. \qquad (18)$$

It should be stressed that values of the thermal coefficients C and L, the dimensionless heat-exchange characteristics Bi and G, as well as the intensity Po of the energy sources, do not enter into (17), (18).

The correlation obtained between the excess heats (18) has been verified experimentally. Eighteen samples of range multipliers of electrical measurement instruments were the subjects tested. The resistor frames were fabricated from Textolite. They were normal in size [3]. The winding was of Ø 0.1, 0.2, or 0.3 mm wire. Four copper–constantan thermocouples, whose lead (Ø 0.1 mm) was bypassed in a circle by the rod of the frame or the surface of the winding, respectively, were mounted on each sample, and their junction was glued with BF glue to improve the contact.

The tests were conducted according to the circuit pictured in Fig. 1. The excess heats were measured at points 1, 2, 3, 4 in the stationary thermal state, where $\Delta T_1 \approx$ 55-95°K for each of the four intensity modes corresponding to excess heat at point 1.

The test excess heats at points 1, 2, 3, 4 and the computed excess heat

$$\Delta T_2 = \frac{\Delta T_1 \Delta T_4}{\Delta T_3} \qquad (19)$$

in Table 1 are presented only for one of the modes of each coil, and the discrepancy between the test and computed values ΔT_2 for all the modes (they are written in the last column in increasing values of the intensity, and are isolated for the cited excess heats in heavy type).

The last column in Table 1 shows that computations of the excess heat at point 2 by means of (19) agree satisfactorily with the experimental results. In the overwhelming majority of cases the discrepancy does not exceed 10%.

TABLE 1. Experimental and Theoretical Excess Heats at Points 1, 2, 3, 4

Frame number	Dimensions, mm			Test $\Delta T_{(ex)}$, °K at points				$\Delta T_{2(19)}$, °K by (19)	$\frac{\Delta T_{2(19)}-\Delta T_{2(ex)}}{\Delta T_{2(ex)}} \cdot 100, \%$
	d	D	h	1	2	3	4		
MN 4073-63									
I-06	8	12	20,5	57,9	61,0	40,0	44,8	64,8	+(6,2; 5,5; 5,8; 7,5)
I-10	10	16	15	57,5	60,2	46,5	48,9	60,4	+(0,3; 0,4; 0,6; 0,0)
I-11	10	16	20	60,3	65,9	45,4	51,3	68,1	+(3,3; 3,3; 3,8; 3,8)
I-14	12	16	44,5	61,2	62,8	43,5	43,2	60,7	—(3,3; 3,6; 3,3; 3,5)
I-20	8	20	19,5	59,7	64,7	47,1	54,5	69,0	+(6,6; 7,0; 7,9; 6,5)
I-21	16	20	19,5	64,5	64,8	66,6	47,9	51,1	+(4,2; 3,7; 4,3; 3,6)
I-22	8	20	26,5	71,9	81,0	51,7	62,9	87,4	+(9,2; 7,9; 9,0; 8,4)
I-23	10	25	11	64,9	70,3	54,7	65,0	77,0	+(9,3; 9,5; 9,4; 10,2)
I-24	10	25	15	70,4	76,9	55,6	67,0	84,8	+(10,4; 10,3; 10,8; 11,0)
I-25	10	25	19,5	73,1	79,9	58,8	66,9	83,1	+(3,1; 4,0; 0,1; 3,7)
I-26	20	25	19,5	76,5	78,6	60,4	62,0	78,5	+(0,3; —0,1; —0,1; —0,2)
I-28	10	25	34,5	76,4	87,0	52,5	62,6	91,0	+(3,8; 3,7; 4,6; 5,4)
I-29	10	32	19,5	81,0	91,7	65,5	80,0	98,9	+(9,3; 7,7; 7,9; 8,2)
I-30	12	32	26,5	75,3	85,7	58,7	70,1	89,9	+(4,2; 5,4; 4,9; 5,7)
I-31	12	32	34,5	93,5	104,7	68,3	81,6	111,7	+(6,4; 5,2; 7,2; 6,7)
I-33	12	40	34,5	99,8	117,7	72,6	91,6	125,9	+(7,6; 8,1; 8,0; 7,0)
III-49	8	16	27	88,7	96,4	64,7	69,8	95,6	—(1,3; 1,6; 1,3; 0,8)
MN 4075-62									
15	8	20	12,5	82,4	90,1	67,0	78,2	96,1	+(6,9; 7,1; 7,2; 6,6)

TABLE 2. Values of the Initial Quantities and Calculated Excess Heats

Sample number	Dimensions, mm			Digital computer computation of ΔT, °K at the points				ΔT, °K by (19)
	d	D	h	1	2	3	4	
1	10	30	32	46,7	51,9	41,3	45,4	51,4
2	10	30	4	30,0	31,8	29,7	31,4	31,7
3	10	18	32	34,2	35,3	31,7	32,6	35,2
4	6	18	20	138,2	153,5	133,2	147,7	153,2

Thermal characteristics: $\alpha_0 = 20$; $\alpha_B = 5$; $\alpha_e = 16$ W/m^2·deg; $c = 400$ J/kg·deg; $\rho = 5000$ kg/m^3; $\lambda_0 = 0.4$ W/m·deg; $q_v = 10^5$ W/m^3; $T_c = 293$°K; in the example $4\alpha_{in} = 0$; $q_v = 5 \cdot 10^5$ W/m^3.

The correlation (18) was also verified for bounded cylindrical tubes with internal energy sources whose temperature field was computed on the "Minsk-1" digital computer by the method of elementary balances. Such a verification was carried out on four bodies with different relations between the dimensions, as well as for differing heat-exchange conditions in one of the samples. The calculated excess heats and initial values are presented in Table 2.

The results of this verification, free of experimental errors, again indicate the correctness of the considered correlation.

NOTATION

$X = x/l$, $Y = y/l$, $Z = z/l$ are the dimensionless coordinates;

$\theta = T/T_c$ is the dimensionless temperature, or the ratio between the temperature examined and the temperature of the medium;

$C(\theta) = c(\theta)\rho(\theta)/c_0$, $L(\theta) = \lambda(\theta)/\lambda_0$ are the dimensionless volume heat conduction and heat conduction, respectively;

$G = \varepsilon_B T_c^3 l/\lambda_0$ is the dimensionless relative characteristic of the radiated heat flux;

α_o, α_{in}, α_e are the heat-exchange coefficients for the outer, inner, and endface surface of the tube, respectively.

Subscript

s denotes phenomena on the body surface.

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